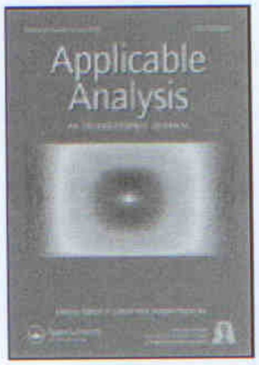

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On the existence of solutions for discrete elliptic boundary value problems

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On the Existence and Stability of Solutions for a System of Elliptic Equations

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Abstract. We show, using a dual variational method developed in the paper, the existence of solutions for a system of Dirichlet problems for partial differential equations involving $p-$ and $q-$Laplacian. The stability of solutions and the existence of positive solutions is considered in the superlinear case.

Mathematics Subject Classification (2000). 35B18.

Keywords. Nonconvex elliptic Dirichlet problems, positive solutions, duality method, variational method.

1. Introduction

The main aim of the paper is to prove the existence of solutions, by a dual variational method which we develop, to the following system of perturbed $q, p-$Laplace equations with Dirichlet boundary data, namely

$$\begin{cases} -\operatorname{div}(k(x)|\nabla u(x)|^{q-2}\nabla u(x)) = F_u(x, u(x), v(x)) & \text{for a.e. } x \in \Omega \\ -\operatorname{div}(l(x)|\nabla v(x)|^{p-2}\nabla v(x)) = F_v(x, u(x), v(x)) & u \in W_0^{1,q}(\Omega),\ v \in W_0^{1,p}(\Omega), \end{cases} \tag{1.1}$$

where $q, p \geq 2$, $k, l \in C^1(\overline{\Omega}, R_+)$ and F_u, F_v denote the derivatives of F with respect to u and v, respectively.

Having proved that the solution, in a sense which we describe below, exists, we are in position to consider the classical problem of stability of solutions which in this case becomes non-trivial due to the fact that the solution is not unique. To provide stability results we consider a family of problems of type (1.1) with nonlinearity depending on a numerical parameter.

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ON POSITIVE SOLUTIONS FOR A CLASS OF QUASILINEAR ELLIPTIC SYSTEMS

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Abstract. We investigate the existence and properties of solutions for a class of systems of Dirichlet problems involving the perturbed phi-Laplace operators. We apply variational methods associated with the Fenchel conjugate. Our results cover both sublinear and superlinear cases of nonlinearities.

1. Introduction

We investigate the existence and properties of positive solutions for the following system of Dirichlet problems involving a class of perturbed phi-Laplace operators:

$$(S_\varphi)\qquad \begin{cases} -\big(l_1(t)\varphi_1\big(z'(t)\big)\big)' = H_z\big(t,z(t),u(t)\big) & \text{in } (0,1),\\ -\big(l_2(t)\varphi_2\big(u'(t)\big)\big)' = H_u\big(t,z(t),u(t)\big) & \text{in } (0,1)\\ z(0)=z(1)=0, \quad u(0)=u(1)=0 \end{cases}$$

with $H_z := \partial H/\partial z$, $H_u := \partial H/\partial u$. We shall discuss (S_φ) under the following basic assumptions:

(A1) The function $H : (0,1)\times(a,c)\times(a,c) \to (0,+\infty)$ with $a<0<b<c$ is continuous and Gateaux differentiable, convex with respect to the pair of

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EIGENVALUE PROBLEMS FOR SINGULAR ODES

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Abstract. We investigate eigenvalue intervals for the Dirichlet problem when the nonlinearity may be singular at $t = 0$ or $t = 1$. Our approach is based on variational methods and cover both sublinear and superlinear cases. We also study the continuous dependence of solutions on functional parameters.

2010 *Mathematics Subject Classification.*

1. Introduction. We investigate the Dirichlet problem

$$\begin{aligned} -u'' &= f(t,u) + \lambda g(t,u) \text{ a.e. in } (0,1) \\ u(0) &= u(1) = 0 \end{aligned} \tag{1}$$

under the following assumptions:

(H1) there exists $d > 0$ such that $f, g : (0,1) \times (-\varepsilon, d+\varepsilon) \to R$ are Caratheodory functions, $f + \lambda g$ is nondecreasing with respect to the second variable in $(-\varepsilon, d+\varepsilon)$, ε is given positive number.

(H2) $\max_{u \in [0,d]} |f(t,u) + \lambda g(t,u)|$ belongs to $L^2(0,1)$; $f(t,0) + \lambda g(t,0) \neq 0$.

In the literature there are many papers devoted to singular second-order differential equations (see [**1, 2, 5, 8, 9**]). Most papers discuss the case when the nonlinearity is positive in a certain neighbourhood of zero and it is sublinear with respect to the second variable at infinity. In this paper, we consider (1) where the right-hand side may be singular at $t = 0$ or $t = 1$. Moreover, our results will cover both sublinear and superlinear cases at zero and/or at infinity. Our approach is based on assumptions on the nonlinearity in the interval $(-\varepsilon, d+\varepsilon)$ only (see Section 2). Assumptions (H1) and (H2) allow us to obtain an existence result for our problem in each nonempty subset X_λ of $H_0^1(0,1) \cap H^2(0,1)$ such that:

(i) X_λ has property (D), namely for each $u \in X_\lambda$ there exists $\overline{u} \in X_\lambda$ such that

$$\begin{aligned} -\overline{u}'' &= f(t,u) + \lambda g(t,u) \text{ a.e. in } (0,1) \\ \overline{u}(0) &= \overline{u}(1) = 0. \end{aligned} \tag{D}$$

(ii) for each $u \in X_\lambda$, $u(t) \in [0,d]$ for all $t \in [0,1]$.

In the next section we shall give two examples of X_λ satisfying (i) and (ii).

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The continuous dependence on parameters of solutions for a class of elliptic problems on exterior domains

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ABSTRACT

The goal of this paper is to discuss the continuous dependence of solutions on functional parameters for the following semilinear elliptic partial differential equation: $\Delta u(x) + \bar{f}(x, u(x), v(\|x\|)) + g(\|x\|)x \cdot \nabla u(x) = 0$, for $x \in \Omega_{r_0} := \{x \in \mathbb{R}^n, n \geq 3, \|x\| > r_0\}$ and $v \in V$, where V stands for some functional space. Our approach covers the case when f may change sign and admits general growth. As an additional result, the characterization of the radius r_0 for which our problem possesses at least one positive evanescent solution in the exterior domain Ω_{r_0} is described and numerically illustrated. Our approach relies on the subsolution and supersolution method and on a lemma due to Noussair and Swanson.

1. Introduction

We investigate the following problem:

$$\begin{cases} \Delta u(x) + \bar{f}(x, u(x), v(\|x\|)) + g(\|x\|)x \cdot \nabla u(x) = 0, & \text{for } x \in \Omega_{r_v} \\ \lim\limits_{\|x\| \to \infty} u(x) = 0, \end{cases} \tag{1.1}$$

where $n \geq 3$, $\|x\| := \sqrt{\sum_{i=1}^{n} x_i^2}$, $\Omega_{r_0} = \{x \in \mathbb{R}^n,\ \|x\| > r_0\}$, for a given number $r_0 > 1$ and $v \in V \subset C((1, +\infty), \mathbb{R}^p)$ $(p \geq 1)$. The Schrödinger equation extended in Problem (1.1) arises in many applications in physics and numerous problems posed in exterior domains: quantum mechanics (Euclidean scalar field equations), physics (nonlinear optics, laser propagation), astrophysics (stellar structure), population ecology and population genetics (logistic type equations), fluvial hydraulics and stream movements of fluids filling some or all of the plane $\mathbb{R}^2$ or the space $\mathbb{R}^3$ (see e.g. [1,2]). When v is fixed, Problem (1.1) has attracted much attention over the last couple of years (see [3–15] and the references therein). One interesting question from a mathematical point of view is the search for solutions that vanish at positive infinity, a phenomenon called evanescence. It is also of great importance to describe the behavior of solutions near infinity. As for the question of the existence of global solutions, it is mainly discussed in [14,16]. The case $g \equiv 0$ corresponding to the Schrödinger equation was considered by Sugie and Yamaoke [17] who proved the existence of positive solutions; the

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Eigenvalue intervals for higher order problems with singular nonlinearities

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ARTICLE INFO

Keywords:
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ABSTRACT

We study eigenvalue intervals for higher order problems with multipoint boundary conditions. We consider the case when the nonlinearity may be singular at $t=0$ or $t=1$. Our approach is based on topological methods and cover both sublinear and superlinear cases. We also study the continuous dependence of solutions on functional parameters.

1. Introduction

We consider the following problem

$$-u^{(n)}(t)=f(t,u(t))+\lambda g(t,u(t)) \quad \text{in } (0,1), \tag{1}$$

$$u(0)=u'(0)=\cdots=u^{(n-2)}(0)=\sum_{i=1}^{m}b_i u^{(n-2)}(\alpha_i)-u^{(n-2)}(1)=0 \tag{2}$$

with fixed numbers $m,n\in\mathbf{N}:=\{1,2,\ldots\}$. We investigate the existence of positive solutions of, (1) and (2) provided that

(f1) *there exists* $0<d$ *such that* $f,g:(0,1)\times[0,d]\to[0,\infty)$, $f,g\in C((0,1)\times[0,d])$, $\int_0^1[f(t,0)+\lambda g(t,0)]dt\neq 0$, $\max_{u\in[0,d]}|f(\cdot,u)+\lambda g(\cdot,u)|\in L^1(0,1)$;
(BC1) $m>1$, $n\geqslant 3$, $\alpha_i\in(0,1)$ for all $i\in\{1,\ldots,m\}$, $0<\alpha_1<\alpha_2<\cdots<\alpha_m<1$;
(BC2) $b_i>0$ for all $i\in\{1,\ldots,m\}$, $\sum_{i=1}^m b_i=1$.

Boundary value problems with various multipoint boundary conditions were discussed in the literature e.g. in [1,3,4] (with boundary condition, (2)) or in [5–7] (with other boundary conditions).

Problems similar to, (1) and (2) were also studied in [2], where the special case of, (1) and (2) was investigated. In [2] using the Krasnoselski's fixed point theorem, the authors obtain some existence and nonexistence results for the nonlinear eigenvalue problem

$$u^{(n)}(t)+\lambda g(t)\mathbf{f}(u(t))=0 \quad \text{in } (0,1) \tag{3}$$

with boundary conditions, (2), for continuous functions $\mathbf{f}:[0,\infty)\to[0,\infty)$ and $g:[0,1]\to[0,\infty)$, $\lambda>0$, b_i satisfying (BC2) and $\frac{1}{2}\leqslant\alpha_1<\alpha_2<\cdots<\alpha_m<1$.

This paper is devoted to the eigenvalue problem for the case when the nonlinearity $f+\lambda g$ (in, (1)) satisfies only local assumptions concerning its smoothness and "growth". It is worth emphasizing that in our approach we need information

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A variational approach to the eigenvalue problem for higher order BVPs with singular nonlinearities

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ABSTRACT

We present a variational approach to the eigenvalue problem for higher order equations with multipoint boundary conditions. Our approach covers the problems in which the nonlinearity may be singular at $t = 0$ or $t = 1$. The results can be applied for both sublinear and superlinear cases. We also describe the numerical characterization of positive solutions.

1. Introduction

The aim of this paper is to investigate the eigenvalue interval for the problem

$$-u^{(n)}(t) = f(t,u(t)) + \lambda g(t,u(t)) \text{ in } (0,1), \tag{1}$$

$$u(0) = u'(0) = \cdots = u^{(n-2)}(0) = \sum_{i=1}^{k} b_i u^{(n-2)}(\alpha_i) - u^{(n-2)}(1) = 0, \tag{2}$$

with fixed numbers $k, n \in \mathbf{N} := \{1,2,\ldots\}$. Recently the research on similar problems has been very active (see e.g. [4,5] (with singular nonlinearities) [1–3,9] (with boundary condition (2)) or in [6–8] (with other boundary conditions). The authors applied various methods, e.g. fixed point theorems or an approximation approach. In this paper we present variational methods based on properties of the Fenchel conjugate (see [10]). It is worth emphasizing that the nonlinearity $f + \lambda g$ (in (1)) satisfies only local assumptions concerning its smoothness and "growth". Moreover our results cover both sublinear and superlinear cases simultaneously, because we need information about the value of $f + \lambda g$ in $[0,d]$ only. In the last section we present examples of higher order problems (1) and (2) and give eigenvalue intervals for them.

Our goal of this paper is to investigate the existence of positive solutions of (1) and (2) provided that

(f1) there exists an open interval $J \subset R$, $0 \in J$, and $0 < d$, $d \in J$ such that $f + \lambda g : (0,1) \times J \to [0,\infty), (0,1) \ni t \mapsto f(t,d) + \lambda g(t,d)$ belongs to $L^2(0,1)$, $\int_0^1 [f(t,0) + \lambda g(t,0)]dt \neq 0$;
(f2) f, g are measurable with respect to the first variable in $(0,1)$ for all $u \in J$ and $f + \lambda g$ is nondecreasing with respect to the second one in J for a.e. $t \in (0,1)$;

(BC1) $k > 1$, $n \geqslant 2$ and even, $\alpha_i \in (0,1)$ for all $i \in \{1,\ldots,k\}$, $0 < \alpha_1 < \alpha_2 < \ldots < \alpha_k < 1$;
(BC2) $b_i > 0$ for all $i \in \{1,\ldots,k\}$, $\sum_{i=1}^{k} b_i = 1$.

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